Pearson Edexcel

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and completing an attempt to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

3. Marks should not be subdivided.

Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- ft – follow through
- cao – correct answer only
- cso - correct solution only. There must be no clear errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent
- dM – dependent method mark
- dp decimal places
- sf significant figures
- $\ast$ The answer is given on the paper – apply cso

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
- If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.

7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.

8. Mark question parts separately unless the mark scheme indicates otherwise.

<u>Usual rules for the method mark for solving a 3 term quadratic:</u>

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to x = ...

2. Formula

Complete attempt to use the correct formula with values for a, b and c leading to x = ... (may be unsimplified).

3. Completing the square (where a = 1; if a ≠ 1 must divide by a first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to x = ...

Question Number	Scheme	Notes	Marks
1	$\mathbf{P}=\begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix}$		
(a)	$\{\det \mathbf{P} =\}\ \ p(p-1)-(-3)(p+1)$ Award for **any** correct expression for det **P**. **If expanded immediately** allow one sign error only in one term of $p^2-p+3p+3$ which can be implied by $p^2\pm 4p+3$ provided this has not come from $ad+bc$		M1
	p^2+2p+3	Correct 3TQ. Terms in any order. Ignore "=0"	A1
			(2)
(b)	If (a) is incorrect, only the M mark is available in (b) and this mark requires working with a 3TQ [from (a) or reattempted] which $\neq 0$ for all real p		
	The M mark requires doing enough work so that only an explanation and conclusion is then needed, e.g., correct numerical expression (or value) for determinant, correct use of formula, correct completion of the square, correct vertex (stated or via graph or differentiation) The A mark always requires an appropriate explanation **and** non-singular/not singular/isn't/can't be singular or "has inverse". Do not accept just "shown" etc.		
	1. Discriminant: d or $b^2-4ac=2^2-4(1)(3)\ \{=4-12=-8\}$ Negative or $d<0$ or $b^2<4ac$, no (real) solution or $\det \mathbf{P}\neq 0$, so non-singular M1: Correct numerical expression (or value) for the discriminant. Allow comparison of b^2 with $4ac$ A1: Explanation and conclusion. **2. Using formula:** $p=\dfrac{-2\pm\sqrt{2^2-4(1)(3)}}{2(1)}\ \left\{=\dfrac{-2\pm\sqrt{-8}}{2}=-1\pm\sqrt{2}\,\mathrm{i}\right\}$ $\{\sqrt{-8}\}$ not real/possible or complex (accept imaginary) or no solution or $\det \mathbf{P}\neq 0$, so non-singular M1: Correct numerical expression for p A1: Explanation and conclusion. **3. Using completing the square:** $p^2+2p+3=(p+1)^2+2$ $\neq 0$ or >0 or □ 2 (not > 2) or min. value = 2 so non-singular Might see $(p+1)^2=-2$ or $p+1=\{\pm\}\sqrt{-2}$ or $\{\pm\}\sqrt{2}\,\mathrm{i}$ or $p=-1\{\pm\}\sqrt{2}\,\mathrm{i}$ followed by not real/possible/complex roots, so non-singular M1: $p^2+2p+3=(p+1)^2+2$ A1: Explanation and conclusion. **Cases continue overleaf**		M1 A1

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>1(b) cont.</td><td colspan="2">4. Differentiation:
$\frac{dy}{dp}=0 \Rightarrow 2p+2=0 \Rightarrow p=-1 \Rightarrow p^2+2p+3=2$
Minimum or "U-shape" or $a > 0$ or appropriate sketch, so non-singular
M1: For −1 and 2
A1: Full explanation and conclusion.
5. Vertex just stated:
Vertex is at $(-1, 2)$
"U-shape" or $a > 0$ or appropriate sketch, so non-singular
M1: Correct vertex. If preceded by $p^2+2p+3=(p+1)^2+2$ award at that point for CTS
A1: Full explanation and conclusion.</td><td></td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td></tr>
<tr><td>(c)</td><td>$\mathbf{P}=\begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix} \Rightarrow$
$\{\mathbf{P}^{-1}=\}\ \frac{1}{"p^2+2p+3"}\begin{pmatrix} p & -p-1 \\ 3 & p-1 \end{pmatrix}$
or $\begin{pmatrix} \frac{p}{"p^2+2p+3"} & \frac{-p-1}{"p^2+2p+3"} \\ \frac{3}{"p^2+2p+3"} & \frac{p-1}{"p^2+2p+3"} \end{pmatrix}$</td><td>M1: $\frac{1}{"p^2+2p+3"}\times$(a changed 2×2 P)
(their changed P must not be or become constant)
<u>OR</u> sight of Adj(P) i.e. $\begin{pmatrix} p & -p-1 \\ 3 & p-1 \end{pmatrix}$ oe
which may be labelled as $\mathbf{P}^{-1}$
A1ft: Correct inverse ft their det P [from part (a) or reattempted] provided it is (and remains) a function of p and accept det P unsimplified. Condone if det P clearly miscopied or rewritten incorrectly e.g.,
$(p+1)^2-2$
Allow $-1(p+1)$ for $-p-1$ but A0 if $-(-3)$ for 3
Isw when a correct or correct ft answer is seen unless a value for p is substituted</td><td>M1A1ft</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 6</td></tr>
</table>

Question Number	Scheme	Notes	Marks
2(a)	$\mathrm{f}(0.3) = ... \ \{3.4563...\}$ $\mathrm{f}(0.4) = -... \ \{4.0615...\}$	Attempts both $\mathrm{f}(0.3)$ and $\mathrm{f}(0.4)$ and achieves a positive **value** for $\mathrm{f}(0.3)$ and a a negative **value** for $\mathrm{f}(0.4)$	M1
	Sign change oe **and** {f(x) is} continuous $\Rightarrow$ root {between x = 0.3 and x = 0.4}	Both $\mathrm{f}(0.3)$= awrt 3.5 or 3.4 (truncated) & $\mathrm{f}(0.4)$ = awrt −4.1 or −4.0 or −4 (truncated), sign change oe, continuity and a minimal conclusion e.g., "root" or "shown". A graph alone is insufficient. Allow "positive, negative" or $\mathrm{f}(0.3) > 0$, $\mathrm{f}(0.4) < 0$ or $\mathrm{f}(0.3)\mathrm{f}(0.4) < 0$ for "sign change".	A1
			(2)
(b)	$\mathrm{f}(x) = x^2 - \dfrac{7x - 4\sqrt{x}}{x^3} = x^2 - 7x^{-2} + 4x^{-2.5}$ $\mathrm{f}'(x) = 2x + 14x^{-3} - 10x^{-3.5}$	Indices must be processed for any marks M1: For $...x^n \to ...x^{n-1}$ at least once A1: 2 correct terms simplified or unsimplified A1: All correct simplified or unsimplified	M1A1A1
	If quotient/product rule used award M1 for any evidence of $...x^n \to ...x^{n-1}$ $\left(\text{e.g., } x^2 \to 2x\right)$. Their **final** expression in these cases must imply 2 correct terms for the first A mark. Quotient rule leads to $2x - \dfrac{x^3\left(7 - 2x^{-\frac{1}{2}}\right) - 3x^2\left(7x - 4x^{\frac{1}{2}}\right)}{\left(x^3\right)^2} = 2x - \dfrac{7x^3 - 2x^{\frac{5}{2}} - 21x^3 + 12x^{\frac{5}{2}}}{x^6} = 2x - \dfrac{-14x^3 + 10x^{\frac{5}{2}}}{x^6}$ Product rule on $x^{-3}\left(7x - 4\sqrt{x}\right)$ leads to $2x - \left[x^{-3}\left(7 - 2x^{-\frac{1}{2}}\right) + \left(-3x^{-4}\right)\left(7x - 4x^{\frac{1}{2}}\right)\right] = 2x - \left[7x^{-3} - 2x^{-\frac{7}{2}} - 21x^{-3} + 12x^{-\frac{7}{2}}\right]$		
			(3)
(c)	$x_1 = 0.3 - \dfrac{\mathrm{f}(0.3)}{\mathrm{f}'(0.3)} = ...$ $\left\{= 0.3 - \dfrac{0.3^2 - \dfrac{7(0.3) - 4\sqrt{0.3}}{0.3^3}}{"2(0.3) + 14(0.3)^{-3} - 10(0.3)^{-3.5}"}\right\}$ $\left\{= 0.3 - \dfrac{3.456304816...}{-157.0821698...} = 0.3 + 0.02200316... = ...\right\}$	Obtains a **value** from an attempt to apply the correct Newton-Raphson formula. Allow slips with substitution/miscopying and may be using an incorrectly simplified $\mathrm{f}(x)$. Implied by awrt 0.32 (0.322003...) <u>even if f '(x) is incorrect</u>. Not implied by real root which is 0.33 or 0.328 (0.3276079...) If not implied and no substitution is seen accept as minimum $"0.3 - \dfrac{\mathrm{f}(0.3)}{\mathrm{f}'(0.3)} = ..."$ but $"x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} = ..."$ is only acceptable if $x_0 = 0.3$ is seen. If only a value is seen it must round to 0.32	M1
	0.322	For awrt 0.322. Must be decimal Ignore further iterations $"\alpha ="$ is not required - just look for awrt 0.322 regardless of how it is labelled	A1
			(2)

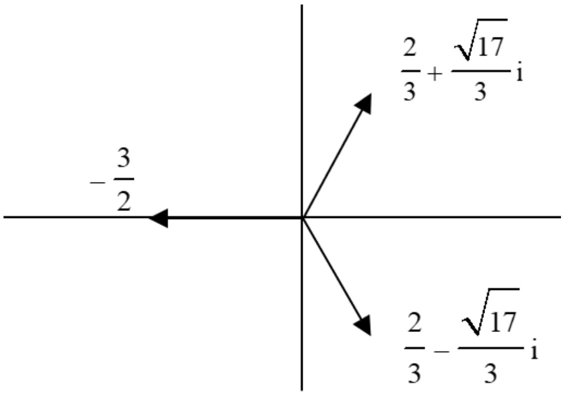
Question Number	Scheme	Notes	Marks
2(d)	$f(1.3) = -0.37613...\ \ f(1.5) = 0.590438...$ Examples: $\frac{"0.590438.."}{1.5-\beta} = \frac{-("-0.37613...")}{\beta-1.3}$ $\frac{1.5-\beta}{\beta-1.3} = \frac{"0.590438.."}{-("-0.37613...")}$ $\frac{1.5-\beta}{1.5-1.3} = \frac{"0.590438..."}{"0.590438..."-("-0.37613...")}$ $\beta = \frac{1.3("0.590438...")-1.5("-0.37613...")}{"0.590438..."-"-0.37613..."}$ $\Rightarrow \beta = ...$	Obtains a negative value for f (1.3) and a positive value for f (1.5), forms a correct equation for these values **and** solves to obtain a value. Apply BOD if only $\lvert f(1.3)\rvert$ and $\lvert f(1.5)\rvert$ seen. Note f(1.5) may be seen as $\frac{64\sqrt{6}-93}{108}$. Accept "f (1.3)" & "f (1.5)" in equation if values for these seen. May use $\frac{af(b)-bf(a)}{f(b)-f(a)}$ oe. Allow e.g., x for β. If their variable denotes e.g., the distance between (1.3, 0) and $(\beta, 0)$ then 1.3 must be added later. Implied by awrt 1.378 (1.3778285...). Not by real root of 1.377 (1.376561...) **Must be using the correct interval.**	M1
	$= 1.378$	For awrt 1.378. Must be decimal. Ignore further iterations "$\beta =$" is not required - just look for awrt 1.378 regardless of how it is labelled	A1
	Alternative via line equation: $y-("-0.37613...") = \frac{"0.590438..."-("-0.37613...")}{1.5-1.3}(x-1.3)$ then $y=0 \Rightarrow x = ...$ M1 for a correct equation with their f (1.3) and f (1.5), setting y = 0 and solving for x. May use $(1.5, "0.590438...")$ as the point. Could also see equivalent attempts using $y = mx + c$ (finds c from a correct equation, puts y = 0 and solves)		
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
3	Score B0 in (a) if the roots $\frac{1\pm\sqrt{14}\,\mathrm{i}}{3}$ are seen and then answers are just written down. The three subsequent method marks require use of the relevant identities. If not, a maximum of 0000010 is likely.		
(a)	$3x^2-2x+5=0\Rightarrow$ $\alpha+\beta=\frac{2}{3},\quad \alpha\beta=\frac{5}{3}$	Both values correct. Consider in order presented if not labelled	B1
			(1)
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(\text{"}\frac{2}{3}\text{"}\right)^2-2\left(\text{"}\frac{5}{3}\text{"}\right)=\ldots$	**Uses** correct identity with their sum and product to obtain a value for $\alpha^2+\beta^2$	M1
	$=-\frac{26}{9}$	Correct value **from correct sum and product**	A1
			(2)
(c)	The work for the first two marks may be embedded within a quadratic expression/equation		
	$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}=\text{"}\frac{2}{3}\text{"}+\frac{\text{"}\frac{2}{3}\text{"}}{\text{"}\frac{5}{3}\text{"}}=\ldots\left(\frac{16}{15}\right)$	Obtains a **value** for the new sum from a <u>correct numerical expression (which could be implied)</u> with their sum and product. Allow use of equivalent numerical expressions following use of e.g. $\frac{\alpha\beta(\alpha+\beta)+\alpha+\beta}{\alpha\beta}$	**1st** M1 (Sum)
	$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}=\alpha\beta+\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{1}{\alpha\beta}$ $=\text{"}\frac{5}{3}\text{"}+\frac{\text{"}-\frac{26}{9}\text{"}}{\text{"}\frac{5}{3}\text{"}}+\frac{1}{\text{"}\frac{5}{3}\text{"}}=\ldots\left(\frac{8}{15}\right)$	Obtains a **value** for the new product via a <u>correct numerical expression (which could be implied)</u> with their sum, product and answer to (b) which may have been reattempted. Allow use of equivalent numerical expressions following use of e.g $\frac{(\alpha\beta)^2+\alpha^2+\beta^2+1}{\alpha\beta}$	**2nd** M1 (Product)
	$x^2-\text{"}\frac{16}{15}\text{"}x+\text{"}\frac{8}{15}\text{"}\ \{=0\}$	Correctly applies x^2 – (their **new** sum)x + their **new** product with **values** to obtain a 3TQ. Not dependent. Accept appropriate values for *p*, *q* and *r* for this mark.	M1
	$15x^2-16x+8=0$	Correct equation – not just values for *p*, *q* and *r*. Terms in any order but must have "=0". Allow any integer multiple. Must come from $\alpha+\beta=\frac{2}{3},\quad \alpha\beta=\frac{5}{3}$ Condone a different variable (e.g., *z*)	A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
4	$\mathrm{f}(z) = 6z^3 + Az^2 + Bz + C$ Condone work in *x* throughout		
(a)	$(z=)\frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i}$	Correct conjugate. Must be seen in (a)	B1
			(1)
(b)	$\left(z - \left(\frac{2}{3} + \frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i}\right)\right) = \ldots \left\{z^2 - \frac{4}{3}z + \frac{7}{3}\right\}$ or e.g. $\alpha+\beta = \frac{2}{3} + \frac{\sqrt{17}}{3}\mathrm{i} + \frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i} = \ldots\left\{\frac{4}{3}\right\}$, $\alpha\beta = \left(\frac{2}{3} + \frac{\sqrt{17}}{3}\mathrm{i}\right)\left(\frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i}\right) = \ldots\left\{\frac{4}{9} + \frac{17}{9} = \frac{7}{3}\right\} \Rightarrow \ldots \left\{z^2 - \frac{4}{3}z + \frac{7}{3}\right\}$ M1: Completes a correct strategy for finding a quadratic factor and obtains a 3TQ with real coefficients. May be slips with the expansion or calculation/forming quadratic but score M0 if the starting point is clearly $\left(z + \left(\frac{2}{3} + \frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z + \left(\frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ A1: Any correct three term quadratic factor. Allow any multiple of $z^2 - \frac{4}{3}z + \frac{7}{3}$ e.g., $3z^2 - 4z + 7$ May see $\left(3z - \left(2 + \sqrt{17}\,\mathrm{i}\right)\right)\left(3z - \left(2 - \sqrt{17}\,\mathrm{i}\right)\right)$ or $(3z-2)^2 = \left(\pm\sqrt{17}\,\mathrm{i}\right)^2 \Rightarrow \ldots \left\{9z^2 - 12z + 21\right\}$		M1A1
	$6\left(z + \frac{3}{2}\right)\left(z^2 - \frac{4}{3}z + \frac{7}{3}\right) = \ldots$ or e.g., $(2z+3)\left(3z^2 - 4z + 7\right) = \ldots$ Multiplies their 3 term quadratic factor with real coefficients (or multiple) by $z + \frac{3}{2}$ (or multiple) to obtain a 4TC with real coefficients so allow $\left(z + \frac{3}{2}\right)\left(z^2 - \frac{4}{3}z + \frac{7}{3}\right) = \ldots\left\{z^3 + \frac{z^2}{6} + \frac{z}{3} + \frac{7}{2}\right\}$		M1
	$\{\mathrm{f}(z) =\}\, 6z^3 + z^2 + 2z + 21$ or $A = 1,\ B = 2,\ C = 21$	A1: Any two correct values for *A*, *B* or *C* (could be embedded) A1: Fully correct expression (ignore an "=0") or three correct values	A1A1
	Note that if the complex factors are not multiplied first e.g., $\left(z - \frac{3}{2}\right)\left(z - \left(\frac{2}{3} + \frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i}\right)\right) = \left(z^2 + \left(\frac{5}{6} - \frac{\sqrt{17}}{3}\mathrm{i}\right)z - 1 - \frac{\sqrt{17}}{2}\mathrm{i}\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ $= z^3 + \frac{1}{6}z^2 + \frac{1}{3}z + \frac{7}{2} \Rightarrow 6z^3 + z^2 + 2z + 21$ Score the first 2 M marks together for obtaining a 4TC with real coefficients. M0M1 is possible e.g., with $\left(z + \frac{3}{2}\right)\left(z + \left(\frac{2}{3} + \frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z + \left(\frac{2}{3} - \frac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ Any correct multiple of the 4TC scores the first A then score as main scheme		
			(5)

Question Number	Scheme/Notes	Marks
4(b)	**Alternative 1**: Substituting to obtain simultaneous equations $z=-\frac{3}{2}\Rightarrow -\frac{81}{4}+\frac{9}{4}A-\frac{3}{2}B+C=0$ $z=\frac{2}{3}\pm\frac{\sqrt{17}}{3}\text{i}\Rightarrow -\frac{188}{9}\mp\frac{10\sqrt{17}}{9}\text{i}+A\left(-\frac{13}{9}\pm\frac{4\sqrt{17}}{9}\text{i}\right)+B\left(\frac{2}{3}\pm\frac{\sqrt{17}}{3}\text{i}\right)+C=0$ $\Rightarrow -\frac{188}{9}-\frac{13}{9}A+\frac{2}{3}B+C=0,\quad \pm\left(-\frac{10\sqrt{17}}{9}+\frac{4\sqrt{17}}{9}A+\frac{\sqrt{17}}{3}B=0\right)$ M1: Substitutes $-\frac{3}{2}$ to obtain an equation and substitutes one of $\frac{2}{3}\pm\frac{\sqrt{17}}{3}$ and equates real and imaginary parts to obtain two further equations. All equations must have real coefficients and each variable must appear in at least one equation. A1: All three correct equations $13A-6B-9C=-188,\ 4A+3B=10,\ 9A-6B+4C=81$ $\Rightarrow A=1,\ B=2,\ C=21$ M1: Solves to obtain real values for *A*, *B* and *C* A1: Two correct values A1: All three correct values **Alternative 2**: Sum/product/pairwise product sum of roots of cubic $\text{sum}=\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}+\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}-\frac{3}{2}=...\left\{-\frac{1}{6}\right\}=-\frac{A}{6}$ $\text{pairwise product sum}=-\frac{3}{2}\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}\right)-\frac{3}{2}\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}\right)+\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}\right)\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}\right)=...\left\{\frac{1}{3}\right\}=\frac{B}{6}$ $\text{product}=-\frac{3}{2}\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}\right)\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}\right)=...\left\{-\frac{7}{2}\right\}=-\frac{C}{6}$ M1: Obtains one equation in *A*, one in *B* and one in *C* all with real coefficients A1: All three correct equations $-\frac{1}{6}=-\frac{A}{6}\Rightarrow A=1,\quad \frac{1}{3}=\frac{B}{6}\Rightarrow B=2,\quad -\frac{7}{2}=-\frac{C}{6}\Rightarrow C=21$ M1: Solves to obtain real values for *A*, *B* and *C* Note that *A*, *B* and *C* would be implied by e.g., $-\frac{1}{6},\ \frac{1}{3},\ -\frac{7}{2}\Rightarrow x^3+\frac{1}{6}x^2+\frac{1}{3}x+\frac{7}{2}$ A1: Two correct values A1: All three correct values If real values for any of the sum/pairwise product sum/product are not explicitly seen allow the M marks if real values for *A*, *B* and *C* (which could be embedded) are obtained. The first A mark would then require all values correct (or a correct cubic) to be awarded. It is possible to e.g., find *A* and *C* as above and then use e.g., $\text{f}\left(-\frac{3}{2}\right)=0$ to determine *B*. In such cases the first M mark is scored when three equations have been attempted.	

4(b)	**Attempts that include long division:** If they find a quadratic factor as in the main scheme e.g., $z^2-\frac{4}{3}z+\frac{7}{3}$ allow M1A1 as before. Dividing it into f(*z*) can lead to equations $B-14+\frac{4}{3}A+\frac{32}{3}=0$ and $C-\frac{7}{3}(A+8)=0$ and these could be used with the equation $-\frac{81}{4}+\frac{9}{4}A-\frac{3}{2}B+C=0$ from using $\mathrm{f}\left(-\frac{3}{2}\right)=0$ or long division by $z+\frac{3}{2}$. Score the next M for obtaining real values for all constants and the A marks as usual. Other attempts including long division that do not find the quadratic factor as per the main scheme we will score as follows. Award the first M1 for credible work to obtain enough equations with real coefficients involving *A*, *B* and *C* so that values could be found, followed by A1 for correct equations. The next M1 is for solving to obtain values for *A*, *B* and *C* and then A1 for two correct values and final A1 for all 3 correct. If divided by $z+\frac{3}{2}$ the quadratic factor is $6z^2+(A-9)z+B-\frac{3}{2}A+\frac{27}{2}$ It is possible to apply the quadratic formula to this and equate the answer to $\frac{2}{3}\pm\frac{\sqrt{17}}{3}\mathrm{i}$ and generate further equations that way. (Long division by complex factors is unlikely but could lead to the other equations in Alt 1) There are potentially a lot of possible precise routes involving long division and coefficient comparison etc. but the above mark scheme principles apply.	

Question Number	Scheme/Notes	Marks
4(c)	 May use points or lines/vectors from origin. Note that B0B1 is not possible. 1st B1: real root plotted on negative *x*-axis and the conjugate pair plotted in Q1 and Q4, roughly aligned vertically. Ignore both axis scales and all labelling. 2nd B1: All roots plotted correctly and labelled. Only consider scale of *x*/real axis so both of the conjugate pair must be **clearly closer** to the *y*/imaginary axis than the real root. Award 2nd B0 if one of the conjugate pair is less than half the distance of the other from the *x*/real axis. If labelled with coordinates they must be the correct way around. Accept $\left(\frac{2}{3}, \pm\frac{\sqrt{17}}{3}\right)$ or $\left(\frac{2}{3}, \pm\frac{\sqrt{17}}{3}\mathrm{i}\right)$ if coordinates are used and accept these coordinates in vector notation or indicated by axis labels. If roots are only indicated by e.g. z_1, z_2, z_3 these must have been identified in (c) or earlier. 2nd B0 if axes labelled the wrong way round. (They do not need to be labelled or could be labelled *x* and *y*).	B1 B1
		(2)
		Total 8

Question Number	Scheme	Notes	Marks
5(a)	$r(r+1)(r+5)=r^3+6r^2+5r$	Correct expansion. May be implied	B1
	$\left\{\sum_{r=1}^{n}(r^3+6r^2+5r)=\sum_{r=1}^{n}r^3+6\sum_{r=1}^{n}r^2+5\sum_{r=1}^{n}r=\right\}\frac{1}{4}n^2(n+1)^2+6\times\frac{1}{6}n(n+1)(2n+1)+\frac{5}{2}n(n+1)$ M1: Having achieved two terms of the correct form from the expansion, replaces at least one of $\sum r^3$, $\sum r^2$ or $\sum r$ correctly A1: Correct expression in any form		M1A1
	$=\frac{1}{4}n(n+1)\left[n(n+1)+4(2n+1)+10\right]\Rightarrow\frac{1}{4}n(n+1)(n^2+9n+14)$ Obtains $\frac{1}{4}n(n+1)[...]$ where ... is a 3TQ in n May be implied by subsequent correct work. Note they might expand first to get e.g., $\frac{1}{4}n(n^3+10n^2+23n+14)$ or $\frac{1}{4}(n^4+10n^3+23n^2+14n)$ or $\frac{1}{4}n^4+\frac{5}{2}n^3+\frac{23}{2}n^2+\frac{7}{2}n$ Allow factor reconstruction after solving. Condone poor algebra but if no 3TQ is seen or the expanded form is wrong and they go straight to an answer it must follow. Expect a full method if e.g., expands or partially expands $\frac{1}{4}n(n+a)(n+b)(n+c)$ and equates coefficients. Requires previous M mark.		**d**M1
	$\frac{1}{4}n(n+1)(n+2)(n+7)$	Correct expression. Not just values. Brackets in any order. Accept $\frac{n}{4}(n+1)(n+2)(n+7)$ or $\frac{n(n+1)(n+2)(n+7)}{4}$	A1
			(5)
(b)	$20\times21\times25+21\times22\times26+...+40\times41\times45$ $=\frac{1}{4}\times40(40+1)(40+2)(40+7)-\frac{1}{4}\times19(19+1)(19+2)(19+7)=...$ or $\frac{1}{4}\times40(41)(42)(47)-\frac{1}{4}\times19(20)(21)(26)=...$ Attempts to find a **value** for $\sum_{r=1}^{40}r(r+1)(r+5)-\sum_{r=1}^{19}r(r+1)(r+5)$ using $\mathrm{f}(40)-\mathrm{f}(19)$. We will allow one slip only (one of the eight components of the products) provided a full substitution as above is seen. **There must be explicit evidence that they are using their result from part (a)**. So do not accept just 809340 – 51870 = ... but allow e.g., $=\frac{1}{4}\times3237360-\frac{1}{4}\times207480=...$ Their part (a) must be of the correct form. Allow if they are using made up values for *a*, *b* and *c*.		M1
	= 757 470	757 470 only. Allow 7.5747×10^5 Isw if subsequently rounded	A1
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$xy=100 \Rightarrow y=100x^{-1} \Rightarrow \frac{dy}{dx}=-100x^{-2}$ or $\Rightarrow x\frac{dy}{dx}+y=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{x}$ or $x=10t, y=\frac{10}{t} \Rightarrow \frac{dy}{dx}=\frac{-10t^{-2}}{10}$ **Any** correct expression for $\frac{dy}{dx}$. Does not need to be in terms of *t*. Allow for a correct $\frac{dx}{dy}=-\frac{x^2}{100}$ or $-\frac{x}{y}$ or $-\frac{100}{y^2}$ May use $-\frac{dx}{dy}$. Condone just $\frac{dy}{dx}$ or $m_{\{T\}}=-\frac{1}{t^2}$.		B1
	$m_T=-\frac{100}{100t^2} \Rightarrow m_N=t^2$ or $m_T=-\frac{\frac{10}{t}}{10t} \Rightarrow m_N=t^2$ or $m_T=-t^{-2} \Rightarrow m_N=t^2$ Correct use of the perpendicular gradient rule to obtain a normal gradient **at** $\left(10t, \frac{10}{t}\right)$ May come directly from $-\frac{dx}{dy}$. Starting with just $m_N=t^2$ scores 0110 maximum.		M1
	$y-\frac{10}{t}=t^2(x-10t)$ or $y=t^2x+c, \frac{10}{t}=t^2\times 10t+c \Rightarrow c=...\left\{\frac{10}{t}-10t^3\right\}$	Correctly forms the normal equation with a changed gradient in terms of *t*. Condone late substitution of $x=10t$ and/or $y=\frac{10}{t}$ into gradient if initial gradient not in terms of *t* (the previous mark is then accessible as is the A1*)	M1
	e.g., $ty-10=t^3x-10t^4$ or $ty=t^3x+10-10t^4 \Rightarrow t^3x-ty=10\left(t^4-1\right)$* Obtains the given answer **with intermediate line** and no errors. Final answer could be reversed and terms/products could be in a different order but must have factorised $10t^4-10$. Allow e.g., t^3x-ty written as $t\left(xt^2-y\right)$ All previous marks are required.		A1*
			(4)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>6(b)</td><td>$x=0\Rightarrow -ty=10\left(t^4-1\right)\Rightarrow y=...\left\{-\frac{10\left(t^4-1\right)}{t}\right\}$</td><td>Substitutes $x = 0$ to find y for Q
May just restate their c from (a). Apply BOD throughout if the minus sign just disappears</td><td>M1</td></tr>
<tr><td></td><td>$\frac{1}{2}\left(\text{"}\frac{10\left(t^4-1\right)}{t}\text{"}\right)\times 10t=750$
May see x-axis intercept also used e.g.,
$\frac{1}{2}\times\text{"}\frac{10\left(t^4-1\right)}{t^3}\text{"}\times\left(\frac{10}{t}+\text{"}\frac{10\left(t^4-1\right)}{t}\text{"}\right)=750$
Allow with modulus signs used</td><td>Correct method for the area of the triangle and sets = 750 or equivalent work
e.g., $\left(\text{"}10t^3-\frac{10}{t}\text{"}\right)\times 10t=1500$
Allow with their y (and possibly their x-axis intercept) and allow for the sign of their y (and/or x) coordinate uncorrected and note that $-10t$ may be used with uncorrected y.
There are no marks if they have Q on the x-axis instead of the y-axis</td><td>M1</td></tr>
<tr><td></td><td colspan="2">"Shoelace" methods only get credit when the determinants are processed. Look for expressions as above.</td><td></td></tr>
<tr><td></td><td colspan="2">We will score the first 2 M marks in this order for this possible variation:
$\frac{1}{2}\left|y_Q\right|\times 10t=750\Rightarrow y_Q=-\frac{150}{t}$ 1st M1: Uses a correct triangle method to find y coord.
$\left(0,\frac{-150}{t}\right)$ in $t^3x-ty=10\left(t^4-1\right)\Rightarrow 150=10\left(t^4-1\right)$ 2nd M1: Subs. into normal equation</td><td></td></tr>
<tr><td></td><td colspan="2">$\frac{1}{2}\left(\frac{10\left(t^4-1\right)}{t}\right)\times 10t=750$ or $\frac{1}{2}\left(10t^3-\frac{10}{t}\right)\times 10t=750$ or $\frac{1}{2}\left(-\frac{10\left(t^4-1\right)}{t}\right)\times -10t=750$ or $\frac{1}{2}\left(\frac{10}{t}-10t^3\right)\times -10t=750$
$\Rightarrow\left\{50t^4-50=750\Rightarrow\right\}t^4=...\{16\}$
Reaches $t^4=...$ (or t^2 = ... or t = ...) from a correct equation. Allow if ... is negative.
If they additionally work with an incorrect equation then ignore this work.
Modulus signs must have been removed although this could happen later in the working, but do be vigilant with attempts where e.g., the "16" has clearly not been obtained appropriately.</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$\left\{\Rightarrow t=\pm 2\Rightarrow\left(10\times\pm 2,\ \frac{10}{\pm 2}\right)\Rightarrow\right\}(20,\ 5),\ \ (-20,\ -5)$ only
A1: One correct pair of coordinates
A1: Both correct and no others including complex solutions.
Allow for x = ..., y = ... but must be clearly paired correctly
Allow $\pm(20,\ 5)$ or $(\pm 20,\ \pm 5)$ Score A1 A0 for e.g., $\left(\pm 20,\ \pm\frac{10}{2}\right)$
Do not accept correct coordinates if they have come from an incorrect equation.
Note that the second pair of coordinates might be deduced.</td><td>A1
A1</td></tr>
<tr><td></td><td colspan="2"></td><td>(5)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 9</td></tr>
</table>

Question Number	Scheme	Notes	Marks
7(i)(a)	A rotation of 240°/$\frac{4\pi}{3}$ (anti/counter clockwise) about/around/at centre (0, 0)/origin/*O* M1: Any rotation. Condone "rotate" for all marks A1: Fully correct description. Condone missing degrees symbol. If direction is not mentioned assume anticlockwise so allow rotation of 120°/$\frac{2\pi}{3}$ **clockwise** about *O*. –120°/ $-\frac{2\pi}{3}$ (anticlockwise) or –240°/ $-\frac{4\pi}{3}$ **clockwise** are also acceptable. Must be a single transformation.		M1A1
			(2)
(b)	$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix. Allow if they omit (b) and this matrix is seen in (c).	B1
			(1)
(c)	$\{\mathbf{C} = \mathbf{AB} =\}\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\text{"}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\text{"}$	Forms a correct product of matrices with their **B** Condone a clear miscopy of **A**	M1
	$\begin{pmatrix} -1 & \frac{\sqrt{3}}{2} \\ -\sqrt{3} & -\frac{1}{2} \end{pmatrix}$	Correct matrix. Any equivalent. No evidence of incorrect method for product or any incorrect matrices.	A1
			(2)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="6">7(ii)(a)</td><td>$\begin{pmatrix} k & -2 \\ -1 & 2k \end{pmatrix}\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} k^2-2k \\ -k+2k^2 \end{pmatrix} \left\{= \begin{pmatrix} 35 \\ 91 \end{pmatrix}\right\}$</td><td>Multiplies to obtain one correct element.
May just see e.g., $\begin{pmatrix} k & -2 \end{pmatrix}\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} k^2-2k \end{pmatrix}$</td><td>M1</td></tr>
<tr><td colspan="2">$k^2-2k-35=0 \Rightarrow k=...$ or $2k^2-k-91=0 \Rightarrow k=...$
Attempts to solve one correct quadratic equation. Usual rules. One root correct if no working. This mark can also be awarded if they solve a correct 3TQ following combining correct equations e.g.,
$3k^2-3k-126\{=k^2-k-42=0\}=0 \Rightarrow k=...\{7, -6\}$
If they eliminate to a correct equation that isn't a 3TQ then one solution must be correct e.g., $3k^2-147=0 \Rightarrow k^2=49 \Rightarrow k=\{\pm\}7$ or $-3k+21=0 \Rightarrow k=7$
If reduced to a linear equation allow all marks otherwise 1100 max if the evidence is that only one quadratic has been solved (e.g., 2 solutions from one equation offered even if one is "rejected" etc. rather than crossed out - ignore all crossed out work)</td><td>M1</td></tr>
<tr><td>$k^2-2k-35=0 \Rightarrow k=...\{-5, 7\}$
and
$2k^2-k-91=0 \Rightarrow k=...\{-\frac{13}{2}, 7\}$
Or one of the above with one of
$3k^2-3k-126=0\ \{\Rightarrow k^2-k-42=0\} \Rightarrow k=...\{7, -6\}$
or $3k^2-147=0\{\Rightarrow k^2=49\} \Rightarrow k=...\{7, -7\}$
Or
$\Rightarrow -3k+21=0 \Rightarrow k=7$

Factorisations shown below</td><td>Attempts to solve two correct quadratic equations (or one correct linear). Usual rules. One root correct if no working. Allow if they obtain 7 from one equation and verify correctly that it works in the second equation (or shows that the other root doesn't work). It is valid to solve one of the equations with a combined equation. Allow if equations not "extracted" from matrices e.g.,
$\begin{pmatrix} k^2-2k \\ -k+2k^2 \end{pmatrix} = \begin{pmatrix} 35 \\ 91 \end{pmatrix}$</td><td>M1</td></tr>
<tr><td colspan="3">$k^2-2k-35=(k+5)(k-7)$, $2k^2-k-91 \Rightarrow (2k+13)(k-7)$, $k^2-k-42 \Rightarrow (k+6)(k-7)$, $k^2-49 \Rightarrow (k+7)(k-7)$</td></tr>
<tr><td colspan="2">$k=7$ following use of 2 correct equations. No incorrect values of k or incorrect factorisations etc seen. No other values offered. Two correct equations (which could be "unextracted") followed by 7 <u>only</u> is sufficient. Condone poor matrix work such as using (k, k) or matrix multiplications written the wrong way around provided correct elements/equations are obtained.</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
<tr><td></td><td colspan="2">Alternative using inverse:
$\mathbf{M}\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} 35 \\ 91 \end{pmatrix} \Rightarrow \begin{pmatrix} k \\ k \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 35 \\ 91 \end{pmatrix}$ and $\mathbf{M}^{-1} = \frac{1}{2k^2-2}\begin{pmatrix} 2k & 2 \\ 1 & k \end{pmatrix} \Rightarrow \begin{pmatrix} k \\ k \end{pmatrix} = \frac{1}{2k^2-2}\begin{pmatrix} 70k+182 \\ 35+91k \end{pmatrix}$
M1: For $70k+182$ or $35+91k$
$k(2k^2-2)=70k+182 \Rightarrow k^3-36k-91\{=(k-7)(k^2+7k+13)\}=0 \Rightarrow k=7\ \{\text{or } \frac{-7\pm\sqrt{3}\mathrm{i}}{2}\}$
$k(2k^2-2)=35+91k \Rightarrow 2k^3-93k-35\{=(k-7)(2k^2+14k+5)\}=0$
$\Rightarrow k=7\ \{\text{or } \frac{-7\pm\sqrt{39}}{2} \text{ or awrt} -0.38 \text{ or } -6.6\}$
M1: Obtains a correct root from a correct equation
M1: Obtains a correct root from two correct equations or verifies as mentioned above
OR $70k+182=35+91k \Rightarrow k=7$
M1 M1: Obtains $k=7$ from this linear equation
A1: $k=7$ and no incorrect values/factorisations etc.
Condone poor matrix work such as using (k, k) or matrix multiplications written the wrong way around provided correct elements/equations are obtained.</td><td></td></tr>
</table>

Question Number	Scheme	Notes	Marks
7(ii)(b)	$\left\{\begin{vmatrix} "7" & -2 \\ -1 & 2\times"7" \end{vmatrix} \Rightarrow \right\}\ "7"\times 2\times"7"-(-1)(-2)\ \{=96\}$	Correct numerical expression for det **M** correct for any of their values of *k*. May be implied by sight of "96"$\times$336 (or e.g. 32256). Allow an invented **value** of *k*	M1
	$\Rightarrow \dfrac{7}{2}\ \left\{\text{or }\ 3.5,\ 3\dfrac{1}{2},\ \dfrac{336}{96}\right\}$	Correct value and <u>no others</u>. Any equivalent	A1
	If points and transformed points are used a correct numerical expression for the area scale factor or its reciprocal must be achieved for the M mark		**(2)**
			Total 11

Question Number	Scheme/Notes	Marks
8	Condone work in n instead of k throughout	
8(i)	$n=1\left\{\text{in}\begin{pmatrix}1-3n & 9n\\ -n & 3n+1\end{pmatrix}\right\}\Rightarrow\begin{pmatrix}1-3 & 9\\ -1 & 3+1\end{pmatrix}=\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ Obtains $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ or $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^1$ with (minimal) substitution seen. One of $1-3$ or $1-3(1)\to-2$ or $9(1)\to 9$ or $-(1)\to-1$ or $3+1$ or $3(1)+1\to 4$ is sufficient. No requirement to say "true" (oe) yet. Ignore further verifications for $n=2$ etc.	B1
	{Assume true for $n = k$:} $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^k=\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}$	
	$\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+1}=\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ or $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}$ Completes an attempt to form $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+1}$ in terms of k	M1
	$=\begin{pmatrix}-2+6k-9k & 9-27k+36k\\ 2k-3k-1 & -9k+12k+4\end{pmatrix}$ or $\begin{pmatrix}-2+6k-9k & -18k+27k+9\\ -1+3k-4k & -9k+12k+4\end{pmatrix}$ or $\begin{pmatrix}-2-3k & 9+9k\\ -k-1 & 3k+4\end{pmatrix}$ Correct unsimplified or simplified matrix with no unexpanded expressions	A1
	$\left\{=\begin{pmatrix}-2-3k & 9+9k\\ -k-1 & 3k+4\end{pmatrix}\right\}=\begin{pmatrix}1-3(k+1) & 9(k+1)\\ -(k+1) & 3(k+1)+1\end{pmatrix}$ Reaches a correct matrix fully in terms of k + 1 (terms in any order and allow for any k + 1 to be written as 1 + k) with no errors. Meet in the middle approaches must be convincing. Requires previous two marks.	A1
	If the result is **true for *n* = *k* then** it is **true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for** (all) ***n***. Correct conclusion/narrative. All the elements in bold should be satisfied. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. "Assume $n = k$" in the narrative followed by "true for $n = k + 1$" in the conclusion plus "true for $n = 1$" and "true for (all) n" is sufficient. For the last statement allow "true for n", "true for $\mathbb{Z}^+$", "true for $\mathbb{N}$" and condone "true for $\mathbb{Z}$", "true for integers", "true for integers after 1" or similar but do not allow "true for all $n\in\mathbb{R}$" or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If the base case work is omitted or wrong in any other way then 01110 is the maximum available.	A1
		(5)
	Note that is valid to e.g., assume true for $n=k+1$ and show true for $n=k+2$: $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+2}=\begin{pmatrix}1-3(k+1) & 9(k+1)\\ -(k+1) & 3(k+1)+1\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}=\begin{pmatrix}-3k-2 & 9k+9\\ -k-1 & 3k+4\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ $=\begin{pmatrix}6k+4-9k-9 & -27k-18+36k+36\\ 2k+2-3k-4 & -9k-9+12k+16\end{pmatrix}=\begin{pmatrix}-3k-5 & 9k+18\\ -k-2 & 3k+7\end{pmatrix}=\begin{pmatrix}1-3(k+2) & 9(k+2)\\ -(k+2) & 3(k+2)+1\end{pmatrix}$ Use Review for any such approaches you are not sure about.	

Question Number	Scheme/Notes	Marks
8(ii)	$u_1 = 1 \quad u_2 = 4 \quad u_{n+2} = 6u_{n+1} - 9u_n \quad n \geqslant 1 \Rightarrow u_n = 3^{n-2}(n+2)$	
	$n = 1 \Rightarrow u_1 = 3^{-1}(2+1) = 1, \quad n = 2 \Rightarrow u_2 = 3^0(2+2) = 4$ Obtains $u_1 = 1$ and $u_2 = 4$ from $u_n = 3^{n-2}(n+2)$ with some substitution seen for both cases although it may be minimal. Look for any numerical expressions that give 1 and 4. No requirement to say "true" (oe) yet. Ignore work for u_3 and beyond	B1
	{Assume true for $n = k$ and $n = k + 1$:} $u_k = 3^{k-2}(k+2)$ and $u_{k+1} = 3^{k-1}(k+3)$	
	$u_{k+2} = 6u_{k+1} - 9u_k = 6 \times 3^{k-1}(k+3) - 9 \times 3^{k-2}(k+2)$ Attempts u_k and u_{k+1} using $u_n = 3^{n-2}(n+2)$ and proceeds to attempt to use recurrence relation to obtain u_{k+2} in terms of k	M1
	$= 6 \times 3^k + 2k \times 3^k - 2 \times 3^k - k \times 3^k$ or e.g., $2 \times 3^k(k+3) - 3^k(k+2)$ Obtains an expression where all terms are multiples of 3^k. Requires previous mark	**d**M1
	$\{= 4 \times 3^k + k \times 3^k = 3^k(k+4)\} \; = 3^{(k+2)-2}((k+2)+2)$ or $3^{k+2-2}(k+2+2)$ Reaches a correct expression in terms of k + 2 with no errors. Meet in the middle approaches must be convincing.	A1
	If the result is **true for *n* = *k* and *n* = *k* + 1 then** shown **true for *n* = *k* + 2**. As the result has been shown to be **true for *n* = 1 and *n* = 2**, then result is **true for** (all) ***n***. Correct conclusion/narrative. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. All the elements in bold should be satisfied. "Assume $n = k$ and $n = k + 1$" in the narrative followed by "true for $n = k + 2$" in the conclusion plus "true for $n = 1$ and $n = 2$" and "true for (all) n" or "true for $n \in \mathbb{Z}^+$" is sufficient. For the last statement allow "true for n", "true for $\mathbb{Z}^+$", "true for $\mathbb{N}$" and condone "true for $\mathbb{Z}$", "true for integers", "true for integers after 1" etc. but do not allow "true for all $n \in \mathbb{R}$" or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If just "$u_1 = 1, u_2 = 4$" is seen this is not sufficient evidence of any attempt to substitute and so the maximum score could only be 01110. The same applies if there are any errors in substitution. However e.g., just e.g., "when $n = 1, u_1 = 1$, when $n = 2, u_2 = 4$" can score 01111 since this implies an attempt to verify the values and no errors are seen.	A1
		(5)
		Total 10
	See overleaf for approaches that assume true for $n = k - 1$ and $n = k$ and show true for $n = k + 1$	

8(ii)	**Note that is valid to e.g., assume true for $n = k-1$ and $n = k$ and show true for $n = k+1$:** $n=1 \Rightarrow u_1 = 3^{-1}(2+1)=1, \quad n=2 \Rightarrow u_2 = 3^0(2+2)=4$ $u_{k-1} = 3^{k-3}(k+1) \qquad u_k = 3^{k-2}(k+2)$ $u_{k+1} = 6u_k - 9u_{k-1} = 6\times3^{k-2}(k+2) - 9\times3^{k-3}(k+1)$ $= 2\times3^{k-1}(k+2) - 3^{k-1}(k+1)$ or e.g., $2k\times3^{k-1} + 4\times3^{k-1} - k\times3^{k-1} - 3^{k-1}$ $= k\times3^{k-1} + 3\times3^{k-1} = 3^{k-1}(k+3) = 3^{(k+1)-2}((k+1)+2)$ or $3^{k+1-2}(k+1+2)$ B1: As main scheme M1: Attempts u_{k-1} and u_k using $u_n = 3^{n-2}(n+2)$ and proceeds to attempt to use recurrence relation to obtain u_{k+1} in terms of k dM1: Obtains an expression where all terms are multiples of 3^{k-1}. Requires previous mark A1: Reaches a correct expression in terms of k + 1 with no errors. A1: If result is **true for *n* = *k* - 1 and *n* = *k* then** shown **true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1 and *n* = 2**, then result is **true for** (all) ***n***. See main scheme for guidance on the last mark. Use Review for any similar approaches you are not sure about.	

Question Number	Scheme	Notes	Marks
9(a)	$y^2=\frac{1}{2}x,\ P\left(\frac{t^2}{8},\frac{t}{4}\right)\Rightarrow y=\frac{1}{\sqrt{2}}\sqrt{x}\Rightarrow\frac{dy}{dx}=\frac{1}{2\sqrt{2}\sqrt{x}}=\frac{\sqrt{2}}{4\sqrt{x}}=\frac{\sqrt{2}}{4\sqrt{\frac{t^2}{8}}}\left\{=\frac{1}{t}\right\}$ or $x=\frac{t^2}{8}, y=\frac{t}{4}\Rightarrow\frac{dy}{dx}=\frac{\frac{1}{4}}{\frac{2}{8}t}\left\{=\frac{1}{t}\right\}$ or $2y\frac{dy}{dx}=\frac{1}{2}\Rightarrow\frac{dy}{dx}=\frac{1}{4y}=\frac{1}{4\left(\frac{t}{4}\right)}\left\{=\frac{1}{t}\right\}$ or $x=2y^2\Rightarrow\frac{dx}{dy}=4y\Rightarrow\frac{dy}{dx}=\frac{1}{4y}=\frac{1}{4\left(\frac{t}{4}\right)}\left\{=\frac{1}{t}\right\}$ Correct $\frac{dy}{dx}$ **in terms of *t*.** Could be unsimplified. Accept just "$\frac{dy}{dx}$ or $m=\frac{1}{t}$"		B1
	$y-\frac{t}{4}=\frac{1}{t}\left(x-\frac{t^2}{8}\right)$ or $\frac{t}{4}=\frac{1}{t}\left(\frac{t^2}{8}\right)x+c\Rightarrow c=\ldots\ \left\{\frac{t}{8}\right\}$	Correct straight line method with an unchanged gradient in terms of *t*. Condone late substitution of *x*/*y* into gradient if initial gradient not in terms of *t* (the first two marks are then accessible and A1* is possible)	M1
	e.g., $8ty-2t^2=8x-t^2$ or $8yt=8x+t^2$ or $y=\frac{1}{t}x+\frac{t}{8}$ $\Rightarrow 8yt-8x=t^2$ *	Obtains the answer via intermediate line and no errors. Accept answer with t^2 on one side and $8ty-8x$ or $8(yt-x)$ or $8(ty-x)$ on the other (these 2 terms in either order). Requires both previous marks.	A1*
			(3)
(b)	$x=0\Rightarrow 8yt=t^2\Rightarrow y=\frac{t}{8}\quad\left\{Q_y=\frac{t}{16}\right\}$	Correct *y* coordinate of *A* Could be unsimplified.	B1 (M1 on ePen)
	{Midpoint of *AP*:} $\left(\frac{0+\frac{t^2}{8}}{2},\ \frac{"\frac{t}{8}"+\frac{t}{4}}{2}\right)\left\{=\left(\frac{t^2}{16},\ \frac{3t}{16}\right)\right\}$	Finds midpoint of *AP* using a fully correct method for **their** *A* which is of the form $(0,\ f(t))$ May be given as $x=\ldots,\quad y=\ldots$	M1
	$\{\text{equation of } l_2:\}\ y-"\frac{3t}{16}"=-t\left(x-"\frac{t^2}{16}"\right)$ or $y=-tx+c\Rightarrow "\frac{3t}{16}"=-t\left("\frac{t^2}{16}"\right)+c\Rightarrow c=\ldots\ \left\{\frac{3t+t^3}{16}\right\}$	Forms the equation of the perpendicular bisector of *AP* correct for **their midpoint of *AP*** and with gradient $-t$ (oe). Not dependent but the coordinates of their midpoint must both be functions of *t*	M1
	$y=\frac{t}{16}\Rightarrow\frac{t}{16}-\frac{3t}{16}=-tx+\frac{t^3}{16}\Rightarrow x=\frac{2+t^2}{16}\Rightarrow 2+256y^2=16x$ or e.g., $\frac{t}{16}=-tx+\frac{3t+t^3}{16}\Rightarrow 1=-16x+3+t^2\Rightarrow -2=-16x+256y^2$ $y-3y=-16y\left(x-\frac{256y^2}{16}\right)\Rightarrow -2=-16\left(x-\frac{256y^2}{16}\right)\ \left\{\Rightarrow -2=-16x+256y^2\right\}$ Any correct 3 term equation (may be factorised) with *t* eliminated		A1
	$y^2=\frac{1}{16}x-\frac{1}{128}$ Correct equation in the correct form with y^2 on its own on one side. Allow $y^2=0.0625x-0.0078125$ or equivalent fractions for α and β in $y^2=\alpha x+\beta$ e.g., $-\frac{2}{256}+\frac{16}{256}x=y^2$ and isw if **subsequent** factorisation/multiplication etc.		A1
			(5)
			Total 8